Kurskod: TATA 54
Provkod: TEN 1

NUMBER THEORY, Talteori 6 hp
March 8, 2012, 14–18.
Matematiska instituitonen, Linköpings universitet.
Examinator: Leif Melkersson
Inga hjälpmedel är tillåtna! (E.g. no pocket calculators are allowed!)
You can write in swedish, if you do this consistently.
You are rewarded at most 3 points for each of the 6 problems.
To get grade 3, 4 or 5, you need respectively 7, 11 and 14 points.

(1) Show that $n^8 \equiv n^2 \pmod{126}$ for all integers n.

(2) (a) Find a primitive root of 25.
(b) Solve the congruence $x^7 \equiv 7 \pmod{25}$.

(3) Is it possible to write the number 1729 as the sum of
(a) two squares of integers
(b) four squares of integers
(c) three squares of integers

(4) Show that 121 is an Euler pseudoprime to the base 3.

(5) (a) Find the continued fraction expansion of $\sqrt{95}$.
(b) Find the least solution in positive integers of the Pell equation $x^2 - 95y^2 = 1$.

(6) How many ordered pairs $(x, y) \in \mathbb{Z} \times \mathbb{Z}$ of integers are there such that $x^2 + y^2 = 63700$