NUMBER THEORY, Talteori 6 hp
August 31, 2013, 14–18.
Matematiska institutionen, Linköpings universitet.
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Inga hjälpmedel är tillåtna!(For example books or pocket calculators are not allowed.)
You may write in swedish, if you do this consistently.
You are rewarded at most 3 points for each of the 6 problems.
To get grade 3, 4 or 5, you need respectively 7, 11 and 14 points.

(1) Has the diophantine equation
$$x^2 + y^2 = 9996$$
any solutions?

(2) Which is the smallest prime factor of the number
$$N = 100! + 1$$

(3) (a) Solve the congruence $x^2 - 2x - 1 \equiv 0 \pmod{7}$.
   (b) Solve the congruence $x^2 - 2x - 1 \equiv 0 \pmod{7^2}$.

(4) (a) Find the simple continued fraction of $\sqrt{12}$.
   (b) Find a rational number r, such that
$$\left| \sqrt{12} - r \right| < \frac{1}{100}$$

(5) (a) Find a primitive root modulo 31.
   (b) Solve the congruence
$$5x^7 \equiv 3 \pmod{31}$$

(6) (a) Find a primitive root of 61.
   (b) Find a primitive root of 7442.