NUMBER THEORY, Talteori 6 hp
March 14, 2013, 14–18
Matematiska institutionen, Linköpings universitet.
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Inga hjälpmedel är tillåtna!(For example no pocketcalculators
are allowed.)
You may write in swedish, if you do this consistently.
You are rewarded at most 3 points for each of the 6 problems.
To get grade 3, 4 or 5, you need respectively 7, 11 and 14 points.

(1) (a) Find the order of 7 modulo 37.
   (b) Find the (least nonnegative) residue when 7^{1000} is divided
       by 37.

(2) Decide if the number n can be written as the sum of two squares
    of integers $n = x^2 + y^2$ and when this is possible how many such
    ordered pairs (x, y) of integers (positive, zero or negative) there
    are, when
    (a) $n = 81000$
    (b) $n = 270$

(3) (a) Compute the Jacobi symbol
$$\left(\frac{18}{143}\right)$$
   (b) Is the congruence $x^2 \equiv 18 \pmod{143}$ solvable?

(4) Find a primitive root modulo 121.

(5) Find the two smallest positive solutions of the Pell equation
    $x^2 - 30y^2 = 1$

(6) Solve the congruence $x^3 + 2x - 7 \equiv 0 \pmod{100}$.