NUMBER THEORY, Talteori 6 hp
August 30, 2014, 14–18.
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Inga hjälpmedel är tillåtna!(For example books or pocket calculators are not allowed!)
You may write in swedish, if you do this consistently.
You are rewarded at most 3 points for each of the 6 problems.
To get grade 3, 4 or 5, you need respectively 7, 11 and 14 points.

(1) What is the last digit in the number $N = 47^{171}$?

(2) Can the number n be written as the sum of two squares of integers when
    (a) $n = 1230$
    (b) $n = 1233$

(3) (a) Compute the Jacobi symbol $(\frac{35}{141})$.

    (b) Does the congruence $x^2 \equiv 35 \pmod{141}$ have a solution?

(4) Show that 8911 is a Carmichial number !
    (Hint: In order to quickly find the prime factorization of 8911 it can be helpful first to factorize 8910 into primes.)

(5) (a) Show that 5 is a primitive root modulo 47.

    (b) Find all positive integers x, such that $5^{3x} \equiv 16 \pmod{47}$.

(6) Let $\sigma(m) = \sum_{d|m} d$ be the sum of divisors function.
    (a) Let $n = 3^k 5^l$, where k, l are positive integers. Show that
$$\frac{\sigma(n)}{2n} < 1$$

    (b) Show that for every prime number $p \geq 5$ and for all positive integers k and l, that the number $n = 3^k p^l$ is not a perfect number, i.e. $\sigma(n) \neq 2n$