

Kurskod: TATA 54

Provkod: TEN 1

NUMBER THEORY, Talteori 6 hp

August 29, 2015, 14–18.

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Examiner: Leif Melkersson

Inga hjälpmedel är tillåtna! (For example books or pocket calculators are not allowed!)

You may write in Swedish, if you do this consistently.

You are rewarded at most 3 points for each of the 6 problems.

To get grade 3, 4 or 5, you need respectively 7, 11 and 14 points.

- (1) Show that 21 is a primitive root modulo 23.
- (2) Can n be written as the sum of the squares of two integers, when
 - (a) $n = 605$
 - (b) $n = 697$
 - (c) $n = 711$
- (3) Does the congruence $x^4 \equiv 4 \pmod{103}$ have a solution?
- (4)
 - (a) Show that 5 is a primitive root modulo 17.
 - (b) Make a table of indices $\text{ind}_5 a$, $a = 1, 2, \dots, 16$.
 - (c) Find all integers $x \geq 0$, such that $8^x + 13 \equiv 0 \pmod{17}$.
- (5) Show that 561 is an Euler pseudoprime to the base 35.
- (6)
 - (a) Find the continued fraction expansion of $\sqrt{7}$.
 - (b) Find a rational number r , such that $|\sqrt{7} - r| < \frac{1}{10^2}$.