(1) (a) How many (incongruent) primitive roots does the prime number 113 have ?

   (b) Show that 2 is not a primitive root modulo 113.

(2) Factorize the Gaussian integer $11 - 8i$ into Gaussian primes !

(3) Factorize the number $5^{12} - 1$ into prime numbers !

(4) (a) Compute the Jacobi symbol

$$\left(\frac{28}{143}\right)$$

   (b) Decide wether the congruence $x^2 \equiv 28 \pmod{143}$ is solvable or not !

(5) (a) Show that the diophantine equation $x^2 - 7y^2 = -1$ has no solutions !

   (b) Let n be a positive integer, such that $p|n$ for some prime number p with $p \equiv 3 \pmod 4$. Show that the diophantine equation $x^2 - ny^2 = -1$ has no solutions !

(6) (a) Show that

$$n \sum_{d|n} d^{p-2} \equiv \sigma(n) \pmod p$$

   for each prime number p, which is not a prime divisor of n. Here $\sigma(n)$ is the sum of the divisors function.

   (b) Show that if n is a perfect number and p is a prime not dividing n, then

$$\sum_{d|n} d^{p-2} \equiv 2 \pmod p$$