NUMBER THEORY, Talteori 6 hp
March 19, 2015, 14–18.
Matematiska institutionen, Linköpings universitet.
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Inga hjälpmedel är tillåtna! (For example books or pocket calculators are not allowed!)
You may write in Swedish, if you do this consistently.
You are rewarded at most 3 points for each of the 6 problems.
To get grade 3, 4 or 5, you need respectively 7, 11 and 14 points.

(1) Find the remainder when 7^{8253}? is divided by 25.

(2) (a) Can the number 1845 be written as the sum of two squares of integers?

   (b) The same question for the number 3510.

   (c) What is the number of ordered pairs $(x, y) \in \mathbb{Z} \times \mathbb{Z}$ of integers, such that $11\,700\,000 = x^2 + y^2$?

(3) Does the congruence $x^2 \equiv 17 \pmod{77}$ have a solution?

(4) (a) Compute the simple continued fraction of $\sqrt{80}$.

   (b) Find the two smallest positive solutions x, y of the diophantine equation $x^2 - 80y^2 = 1$

(5) (a) Show that 6 is a primitive root modulo 41.

   (b) Find a primitive root modulo 82.

(6) (a) What is the largest order of an integer modulo 77.

   (b) Find an integer which has the largest possible order modulo 77.