Kurskod: TATA 54
Provkod: TEN 1

NUMBER THEORY, Talteori 6 hp
June 9, 2016, 08–12.
Matematiska institutionen, Linköpings universitet.
Examiner: Leif Melkersson
Inga hjälpmedel är tillåtna! (For example books or pocket calculators are not allowed!)
You may write in Swedish, if you do this consistently.
You are rewarded at most 3 points for each of the 6 problems.
To get grade 3, 4 or 5, you need respectively 7, 11 and 14 points.

(1) Show that $n^{21} \equiv n^3 \pmod{108}$ for all integers n.

(2) Can n be written as the sum of two squares of integers, if
   (a) $n = 4949$
   (b) $n = 3069$
   (c) $n = 100\,000\,000\,003$

(3) Show that 3751 is a pseudoprime to the base 3.

(4) (a) Compute the Jacobi symbol

$$\left(\frac{4036}{2013}\right)$$

   (b) Does the congruence $x^2 \equiv 4036 \pmod{2013}$ have a solution?

(5) (a) Expand $\sqrt{15}$ into a continued fraction.
   (b) Does the diophantine equation $x^2 - 15y^2 = -1$ have a solution.

(6) (a) Find $\operatorname{ord}_{43} 2$, the order of 2 modulo 43.
   (b) Show that 3 is a primitive root modulo 43.
   (c) Show that 3 is a primitive root modulo 43^2.