Kurskod: TATA 54
Provkod: TEN 1

NUMBER THEORY, Talteori 6 hp
August 27, 2016, 14–18.
Matematiska institutionen, Linköpings universitet.
Examiner: Jan Snellman
Inga hjälpmedel är tillåtna! (For example books or pocket calculators are not allowed!)
You may write in Swedish, if you do this consistently.
You are rewarded at most 3 points for each of the 6 problems.
To get grade 3, 4 or 5, you need respectively 7, 11 and 14 points.

(1) Find the remainder, when 7^{1242} is divided by 75.

(2) Decide if there exists an integer x, such that $x^2 \equiv 6 \pmod{437}$ or not.

(3) Find all integers x, such that $f(x) \equiv 0 \pmod{49}$, where $f(x) = x^4 + x + 3$

(4) Find the two smallest pairs (x, y) of positive integers solving the diophantine equation $x^2 - 101\,y^2 = -1$.

(5) (a) Compute $\mathrm{ord}_{73}\, 2$.

(b) Find a primitive root modulo 73.

(6) Find all positive integers n, such that $\varphi(n) = 500$. (Hint: If the prime number p divides n, then $p - 1 \mid \varphi(n)$.)