Number theory, Talteori 6hp, Kurskod TATA54, Provkod TEN1
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All problems are worth 3 points. To receive full points, a solution needs to be complete. Prove your assertions, indicate which theorems from the textbook that you have used, and include all auxillary calculations.

No aid, no calculators, tables, nor textbooks.

3:8p 4:13p 5:18p

1) If a, b are relatively prime positive integers, and $ab = c^n$, with n, c positive integers, show that there exists positive integers d, e such that $a = d^n$ and $b = e^n$.

2) Find all solutions in positive integers to the Diophantine equation
$$\frac{1}{x} + \frac{1}{y} = \frac{1}{7}$$

3) For which positive integers n does it hold that $3\phi(n) = \phi(3n)$, where ϕ denotes the Euler phi-function?

4) Calculate $\mu(n)\mu(n+1)\mu(n+2)\mu(n+3)$ for all positive integers n; μ is the Möbius function.

5) Let $p \geq 7$ be a prime. Show that there exist a positive quadratic residue n of p such that $n+1$ is a quadratic residue of p as well.

6) Given that
$$\sqrt{17} = [4; \overline{8}] = 4 + \cfrac{1}{8 + \cfrac{1}{8 + \cfrac{1}{\ddots}}},$$
find the continued fraction expansion of $\frac{1}{\sqrt{17}}$ and $-\frac{1}{\sqrt{17}}$.

7) Show that the integer $m > 2$ has a primitive root if and only if the congruence $x^2 \equiv 1 \mod m$ has precisely the solutions $x \equiv \pm 1 \mod m$.

Hint: Recall that if $k \geq 3$ *is an integer, then* 5 *has (multiplicative) order* 2^{k-2} *modulo* 2^k.